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Balance equations for electron transport in an arbitrary energy band driven by an intense terahertz field. Application to superlattice miniband transport

X.L. Lei[1,2,a] and H.L. Cui[3]

[1] China Center of Advanced Science and Technology (World Laboratory), PO Box 8730, Beijing 100080, P.R. China
[2] State Key Laboratory of Functional Materials for Informatics, Shanghai Institute of Metallurgy, Chinese Academy of Sciences, 865 Changning Road, Shanghai 200050, P.R. China
[3] Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA

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Abstract. We suggest a balance-equation approach to hot-electron transport in a single arbitrary energy band subject to an intense radiation field of terahertz (THz) frequency, including all the multiphoton emission and absorption processes and taking account of realistic scatterings due to impurities and phonons. This approach, which allows one to calculate THz-driving, time-averaging transport based on a set of time-independent equations, provides a convenient method to study the effect of an intense THz electric field on carrier transport in a nonparabolic energy band. As an example, these fully three-dimensional, acceleration- and energy-balance equations are applied to the discussion of superlattice miniband transport at lattice temperature $T = 77$ and 300 K driven by the THz radiation field of varying strengths. It is shown that the current through a dc biased miniband superlattice is greatly reduced by the irradiation of an intense THz electric field.

PACS. 72.30.+q High-frequency effects; plasma effects – 73.50.Mx High-frequency effects; plasma effects – 72.20.Ht High-field and nonlinear effects

1 Introduction

Recently, the nonlinear dynamics of an electron gas driven by intense terahertz (THz) electric fields has become a central focus of many experimental and theoretical studies in the literature [1–12]. Among many interesting phenomena that have been reported, the effect of a THz radiation on electron transport in two-dimensional (2D) semiconductors and superlattices has attracted much attention. It was reported experimentally that as in a quasi-two-dimensional electron system in low temperature [3,4], the current through a dc biased GaAs/AlAs miniband superlattice is also greatly reduced at room temperature when the system is exposed to an intense radiation field of THz frequency [6,7].

Theoretically, these can be referred to as the response of the electron gas to a time-dependent electric field consisting of a dc component and a large-amplitude high-frequency sinusoidal component. Most of the existing theoretical treatments [9–12] of this issue were essentially one-dimensional in nature and made use of the relaxation time approximation. Despite the fact that the basic physical features of Bragg-diffraction-related phenomena follow from a one-dimensional miniband structure, carrier scatterings by impurities, by phonons and among themselves make the problem truly three dimensional (3D) and may lead to results that are significantly different from those of one-dimensional models [13]. On the other hand, although the time-dependent calculation based on the three-dimensional balance equation [14] is a reliable one to investigate the superlattice response to an alternating electric field from low to medium-high frequency, it has to follow the time-variation of the large amplitude THz field and thus is rather time consuming. Furthermore, this approach has an upper limit of the applicable frequency around 2 THz. A different balance-equation method, which includes accurate microscopic treatments of impurity and phonon scatterings and allows one to calculate THz-driving dc transport based on a set of time-independent rather than time-dependent equations, has recently been developed for systems with parabolic energy dispersion and successfully applied to two-dimensional electron system [15]. However, since the major features of superlattice vertical conduction (*e.g.* negative differential mobility) come from the nonparabolicity of the miniband and related Bragg scattering, to examine the effect of a THz radiation on superlattice miniband conduction we have to develop a method capable of dealing with arbitrary energy dispersion and including the effect of Bragg scattering.

[a] e-mail: `xllei@itsvr.sim.ac.cn`

2 Electrons in an general energy band driven by a uniform time-dependent field

We consider a system consisting of N electrons moving in a single energy band. They are scattered by phonons and by randomly distributed impurities. If the single electron state is described by a lattice wave vector $\mathbf{k}$ in a Brillouin zone with the wave function $\psi_{\mathbf{k}}(\mathbf{r})$ and energy $\varepsilon(\mathbf{k})$, the effective Hamiltonian of the electron-phonon system can be written in the form

$$H = \sum_j \varepsilon(\mathbf{p}_j) + H_{ei} + H_{ep} + H_{ph}, \tag{1}$$

where $\mathbf{p}_j$ is the momentum operator of jth electrons. The phonon Hamiltonian H_{ph}, the electron-impurity and electron-phonon couplings, H_{ei} and H_{ep}, are given respectively by

$$H_{ph} = \sum_{\mathbf{q},\lambda} \Omega_{\mathbf{q}\lambda} b^{\dagger}_{\mathbf{q}\lambda} b_{\mathbf{q}\lambda}, \tag{2}$$

$$H_{ei} = \sum_{\mathbf{q},a} u(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}_a} \rho_{\mathbf{q}}, \tag{3}$$

$$H_{ep} = \sum_{\mathbf{q},\lambda} M(\mathbf{q},\lambda) \phi_{\mathbf{q}\lambda} \rho_{\mathbf{q}}. \tag{4}$$

In these equations, $b^{\dagger}_{\mathbf{q}\lambda}$ $(b_{\mathbf{q}\lambda})$ is the creation(annihilation) operator of phonon with wavevector $\mathbf{q}$ in branch λ having frequency $\Omega_{\mathbf{q}\lambda}$ and $\phi_{\mathbf{q}\lambda} = b_{\mathbf{q}\lambda} + b^{\dagger}_{-\mathbf{q}\lambda}$ is the phonon field operator, $\mathbf{r}_a$ stands for the impurity position, $u(\mathbf{q})$ and $M(\mathbf{q},\lambda)$ are the Fourier representations of the impurity potential and the electron-phonon coupling matrix element, and

$$\rho_{\mathbf{q}} = \sum_j e^{i\mathbf{q}\cdot\mathbf{r}_j} \tag{5}$$

stands for the electron density operator.

When a uniform dc (or slowly varying) electric field $\mathbf{E}_0$ and a uniform sinusoidal radiation field of frequency ω and amplitude $\mathbf{E}_\omega$,

$$\mathbf{E}(t) = \mathbf{E}_0 + \mathbf{E}_\omega \sin(\omega t), \tag{6}$$

are applied in the 2D plane, we can describe this electric field by means of a vector potential $\mathbf{A}(t)$ and a scalar potential $\varphi(\mathbf{r})$ of the form

$$\mathbf{A}(t) = (\mathbf{E}_\omega/\omega)\cos(\omega t), \tag{7}$$

$$\varphi(\mathbf{r}) = -\mathbf{r}\cdot\mathbf{E}_0. \tag{8}$$

Under the influence of such an electric field the Hamiltonian of the system becomes

$$H = H_e(t) + H_{ph} + H_I \tag{9}$$

with $H_I \equiv H_{ei} + H_{ep}$, and

$$H_e(t) = \sum_j \left[\varepsilon\left(\mathbf{p}_j - e\mathbf{A}(t)\right) + \varphi(\mathbf{r}_j)\right]. \tag{10}$$

Following the procedure in reference [16], we define the center-of-mass (CM) coordinate $\mathbf{R}$ by

$$\mathbf{R} = \frac{1}{N}\sum_j \mathbf{r}_j. \tag{11}$$

The rate of change of $\mathbf{R}$, *i.e.* the CM velocity, is given by

$$\begin{aligned}\mathbf{V} \equiv \dot{\mathbf{R}} = -i[\mathbf{R},H] &= \frac{1}{N}\sum_j \mathbf{v}(\mathbf{p}_j - e\mathbf{A}(t))\\ &= \frac{1}{N}\sum_{\mathbf{k},\sigma} \mathbf{v}(\mathbf{k} - e\mathbf{A}(t)) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma},\end{aligned} \tag{12}$$

where

$$\mathbf{v}(\mathbf{k}) \equiv \nabla\varepsilon(\mathbf{k}) \tag{13}$$

is the velocity function, $c^{\dagger}_{\mathbf{k}\sigma}$ $(c_{\mathbf{k}\sigma})$ are creation (annihilation) operators of electron in the lattice wavevector representation, and the sum $\mathbf{k}$ runs over a Brillouin zone in the $\mathbf{k}$ space. This velocity can be split into a slowing-varying part $\mathbf{V}_0$ and a rapidly oscillating part $\mathbf{V}_{\mathrm{A}}$ due to the high-frequency field,

$$\mathbf{V} = \mathbf{V}_0 + \mathbf{V}_{\mathrm{A}}, \tag{14}$$

with

$$\mathbf{V}_0 = \frac{1}{N}\sum_{\mathbf{k},\sigma} \mathbf{v}(\mathbf{k}) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}. \tag{15}$$

We calculate the rate of change of the slowly-varying velocity,

$$\begin{aligned}\frac{d\mathbf{V}_0}{dt} &= -i[\mathbf{V}_0, H]\\ &= \frac{e\mathbf{E}}{N}\cdot\sum_{\mathbf{k},\sigma}\nabla\nabla\varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}\\ &\quad -\frac{i}{N}\sum_{\mathbf{k},\mathbf{q},a} u(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}_a}[\mathbf{v}(\mathbf{k}+\mathbf{q}) - \mathbf{v}(\mathbf{k})]\rho_{\mathbf{kq}}\\ &\quad -\frac{i}{N}\sum_{\mathbf{k},\mathbf{q},\lambda} M(\mathbf{q},\lambda)\phi_{\mathbf{q}\lambda}[\mathbf{v}(\mathbf{k}+\mathbf{q}) - \mathbf{v}(\mathbf{k})]\rho_{\mathbf{kq}}.\end{aligned} \tag{16}$$

We can also calculate the rate of change of the energy h_s determined by

$$h_s \equiv \frac{1}{N}\sum_{\mathbf{k},\sigma}\varepsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}, \tag{17}$$

yielding

$$\begin{aligned}\frac{dh_s}{dt} &= -i[h_s, H]\\ &= e\mathbf{E}\cdot\mathbf{V}_0 - \frac{i}{N}\sum_{\mathbf{k},\mathbf{q},a} u(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}_a}[\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k})]\rho_{\mathbf{kq}}\\ &\quad -\frac{i}{N}\sum_{\mathbf{k},\mathbf{q},\lambda} M(\mathbf{q},\lambda)\phi_{\mathbf{q}\lambda}[\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k})]\rho_{\mathbf{kq}}.\end{aligned} \tag{18}$$

$$\sum_{n=-\infty}^{\infty} J_n^2([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega)\, e^{i[\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})](t-t')}\, e^{-in\omega(t-t')} + \sum_{m\neq 0} e^{-im\omega t}$$
$$\times\left[\sum_{n=-\infty}^{\infty} J_n([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega)\, J_{n-m}([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega) e^{i[\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})](t-t')}\, e^{-in\omega(t-t')}\right]. \tag{31}$$

Here $\rho_{\mathbf{kq}}$ is the lattice wavevector representation of the electron density operator:

$$\rho_{\mathbf{q}} = \sum_{\mathbf{k}} \rho_{\mathbf{kq}}, \tag{19}$$

$$\rho_{\mathbf{kq}} = \sum_{\sigma} g(\mathbf{k},\mathbf{q}) c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma}, \tag{20}$$

with $g(\mathbf{k},\mathbf{q})$ being a form factor determined by the electron wave function [16].

To calculate the statistical average of a physical quantity we need the density of matrix of the transport state, which can be solved from the Liouville equation by starting from a parametrized initial state at time $t=-\infty$, in which the phonon system is in equilibrium at the lattice temperature T and the relative electron system is in equilibrium at an electron temperature T_e with a shifted lattice wavevector $\mathbf{p}_d$:

$$\hat\rho|_{t=-\infty} = \hat\rho_0 = \frac{1}{Z} e^{-H_{er}/T_e} e^{H_{ph}/T}, \tag{21}$$

with

$$H_{er} = \sum_j \bar\varepsilon(\mathbf{p}_j) = \sum_j \varepsilon(\mathbf{p}_j - \mathbf{p}_d). \tag{22}$$

Such a choice of the initial state was successfully used in the balance-equation approach to transport in systems with high carrier-density having an arbitrary energy dispersion without radiation field [16,17]. In this paper we are still concerned mainly with such a kind of electron-phonon systems, but under the influence of a high-frequency radiation field.

In the presence of the radiation field, the zero-order Hamiltonian of the electron-phonon system $H_0(t) = H_e(t) + H_{ph}$ is time-dependent. Nevertheless, the density matrix can be obtained to the linear order in H_I, and the statistical average of a dynamical variable O can be written in the form

$$\langle O\rangle = \langle O\rangle_0 - i\int_{-\infty}^{t} dt' \langle [H_I(t'), O(t)]\rangle_0, \tag{23}$$

where $\langle\cdots\rangle_0$ stands for the average with respect to the initial density matrix $\hat\rho_0$, and, for any operator O, $O(t)$ is defined as

$$O(t) \equiv U_0^\dagger(t)\, O\, U_0(t), \tag{24}$$

where the evolution operator $U_0(t)$ obeys the equation

$$i\frac{d}{dt}U_0(t) = H_0(t)U_0(t) \tag{25}$$

and the condition $U_0(0)=1$.

3 Acceleration and energy balance equations

The acceleration and energy balance equations are obtained by taking the statistical average of equations (16, 18) to the linear order in H_I according to equation (23). Note that we have

$$\rho_{\mathbf{q}}(t) = \sum_{\mathbf{k},\sigma} g(\mathbf{k},\mathbf{q}) e^{i[\mathcal{S}_{\mathbf{k}+\mathbf{q}}(t)-\mathcal{S}_{\mathbf{k}}(t)]} c^\dagger_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma}, \tag{26}$$

with

$$\mathcal{S}_{\mathbf{k}}(t) = \int_0^t \varepsilon(\mathbf{k} - e\mathbf{A}(\tau))d\tau. \tag{27}$$

Furthermore, we make the following approximation in the exponential factor in equation (26), which is valid for short-period superlattices subject to a high frequency but not too strong radiation field,

$$\mathcal{S}_{\mathbf{k}+\mathbf{q}}(t) - \mathcal{S}_{\mathbf{k}}(t) = [\varepsilon(\mathbf{k}+\mathbf{q}) - \varepsilon(\mathbf{k})]t - [\mathbf{v}(\mathbf{k}+\mathbf{q}) - \mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega \sin(\omega t), \tag{28}$$

where

$$\mathbf{e}_\omega \equiv e\mathbf{E}_\omega/\omega^2. \tag{29}$$

With this approximation, we have a factor of the form

$$\int_{-\infty}^{t} dt' e^{i[\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})](t-t')} e^{-i[\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega[\sin(\omega t)-\sin(\omega t')]} \tag{30}$$

in the expressions for the frictional forces and energy-transfer rates. Using the equality of the Bessel functions,

$$e^{-iz\sin x} = \sum_{n=-\infty}^{\infty} J_n(z) e^{-inx},$$

we can rewrite the exponential factor in equation (30) as a sum of two terms:

See equation (31) above.

The first term is a function of $(t-t')$ only, as is the rest parts of the integrands in the frictional acceleration and the energy transfer rate. Thus, after the integration over t', it yields a contribution no longer dependent on t. The second term appears to be rapidly oscillating at fundamental frequency ω and its harmonics, since the integration over t' renders its inner part (inside the bracket) a finite constant value while leaving the outer oscillatory factor

$$s_p = \frac{2\pi n_{\rm i}}{N}\sum_{\mathbf{k},\mathbf{q}}|u(\mathbf{q})|^2|g(\mathbf{k},\mathbf{q})|^2\sum_{n=-\infty}^{\infty}J_n^2\left([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega\right)n\omega$$
$$\times\,\delta\left(\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})-n\omega\right)[f(\bar\varepsilon(\mathbf{k}),T_e)-f(\bar\varepsilon(\mathbf{k}+\mathbf{q}),T_e)]$$
$$+\frac{4\pi}{N}\sum_{\mathbf{k},\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2|g(\mathbf{k},\mathbf{q})|^2\sum_{n=-\infty}^{\infty}J_n^2\left([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega\right)n\omega\delta\left(\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})+\Omega_{\mathbf{q}\lambda}+n\omega\right)$$
$$\times\,[f(\bar\varepsilon(\mathbf{k}),T_e)-f(\bar\varepsilon(\mathbf{k}+\mathbf{q}),T_e)]\left[n\left(\frac{\Omega_{\mathbf{q}\lambda}}{T}\right)-n\left(\frac{\bar\varepsilon(\mathbf{k})-\bar\varepsilon(\mathbf{k}+\mathbf{q})}{T_e}\right)\right] \quad (41)$$

intact. As long as one measures the quantities which are averaged over a time interval much longer than the period of the radiation field, the contribution of the second term is irrelevant. Furthermore, after a transient time period the system should arrive at a time-dependent (oscillating) steady state, in which the time average (over a time interval much longer than the period of the radiation field) of the statistical expectations of $d\mathbf{V}_0/dt$ and dh_s/dt vanish. Therefore, we are left with the time-independent acceleration and energy balance equations for the steady-state transport under the THz drive.

$$e\mathbf{E}_0\cdot\mathcal{K}+\mathbf{A}_i+\mathbf{A}_p=0, \quad (32)$$

and

$$e\mathbf{E}\cdot\mathbf{v}_0-w+s_p=0. \quad (33)$$

Here, we have identified

$$\mathbf{v}_0=\langle\mathbf{V}_0\rangle=\frac{2}{N}\sum_{\mathbf{k}}\mathbf{v}(\mathbf{k})f(\bar\varepsilon(\mathbf{k}),T_e) \quad (34)$$

as the average drift velocity of the system, and have denoted the inverse effective mass tensor

$$\mathcal{K}=\langle\hat{\mathcal{K}}\rangle=\frac{2}{N}\sum_{\mathbf{k}}\nabla\nabla\varepsilon(\mathbf{k})f(\bar\varepsilon(\mathbf{k}),T_e), \quad (35)$$

where

$$f(\varepsilon,T_e)=\{\exp[(\varepsilon-\mu)/T_e]+1\}^{-1} \quad (36)$$

is the Fermi function at the electron temperature T_e, and μ is the chemical potential which should be determined by the total number of electrons, N, according to

$$N=2\sum_{\mathbf{k}}f(\varepsilon(\mathbf{k}),T_e). \quad (37)$$

In equations (32–33),

$$\mathbf{A}_i=\frac{2\pi n_{\rm i}}{N}\sum_{\mathbf{k},\mathbf{q}}|u(\mathbf{q})|^2|g(\mathbf{k},\mathbf{q})|^2[\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]$$
$$\times\sum_{n=-\infty}^{\infty}J_n^2\left([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega\right) \quad (38)$$
$$\times\delta\left(\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})-n\omega\right)[f(\bar\varepsilon(\mathbf{k}),T_e)-f(\bar\varepsilon(\mathbf{k}+\mathbf{q}),T_e)]$$

and

$$\mathbf{A}_p=\frac{4\pi}{N}\sum_{\mathbf{k},\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2|g(\mathbf{k},\mathbf{q})|^2[\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]$$
$$\times\sum_{n=-\infty}^{\infty}J_n^2\left([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega\right)$$
$$\times\delta\left(\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})+\Omega_{\mathbf{q}\lambda}+n\omega\right)$$
$$\times[f(\bar\varepsilon(\mathbf{k}),T_e)-f(\bar\varepsilon(\mathbf{k}+\mathbf{q}),T_e)]$$
$$\times\left[n\left(\frac{\Omega_{\mathbf{q}\lambda}}{T}\right)-n\left(\frac{\bar\varepsilon(\mathbf{k})-\bar\varepsilon(\mathbf{k}+\mathbf{q})}{T_e}\right)\right] \quad (39)$$

are the frictional accelerations due to impurity (with density $n_{\rm i}$) and phonon scatterings,

$$w=\frac{4\pi}{N}\sum_{\mathbf{k},\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2|g(\mathbf{k},\mathbf{q})|^2\Omega_{\mathbf{q}\lambda}$$
$$\times\sum_{n=-\infty}^{\infty}J_n^2\left([\mathbf{v}(\mathbf{k}+\mathbf{q})-\mathbf{v}(\mathbf{k})]\cdot\mathbf{e}_\omega\right)$$
$$\times\delta\left(\varepsilon(\mathbf{k}+\mathbf{q})-\varepsilon(\mathbf{k})+\Omega_{\mathbf{q}\lambda}+n\omega\right)$$
$$\times[f(\bar\varepsilon(\mathbf{k}),T_e)-f(\bar\varepsilon(\mathbf{k}+\mathbf{q}),T_e)]$$
$$\times\left[n\left(\frac{\Omega_{\mathbf{q}\lambda}}{T}\right)-n\left(\frac{\bar\varepsilon(\mathbf{k})-\bar\varepsilon(\mathbf{k}+\mathbf{q})}{T_e}\right)\right] \quad (40)$$

is the (per carrier) energy-transfer rate from the electron system to the phonon system, and

See equation (41) above

is the (per carrier) rate of energy the electron system gains from the radiation field through the multiphoton (absorption and emission) process in associate with electron intraband transitions. In the above balance equations, the average drift velocity $\mathbf{v}_0$, the inverse effective mass tensor $\mathcal{K}$, the frictional accelerations $\mathbf{A}_i$ and $\mathbf{A}_p$, the electron energy-loss rate w and the energy-gain rate s_p, are functions of $\mathbf{p}_d$ and T_e. $\mathbf{A}_i$, $\mathbf{A}_p$, w and s_p also depend on the amplitude E_ω and the frequency ω of the radiation field. Thus, the effects of a radiation field on carrier transport are included. For a system of known energy dispersion $\varepsilon(\mathbf{k})$, one can, from balance equations (32, 33), determine $\mathbf{p}_d$ and T_e (thus the average drift velocity $\mathbf{v}_0$), when $\mathbf{E}_0$, $\mathbf{E}_\omega$ and ω are given.

In the case without a radiation field ($E_\omega=0$), we have $s_p=0$, and $\mathbf{A}_i$, $\mathbf{A}_p$ and w reduce to the corresponding

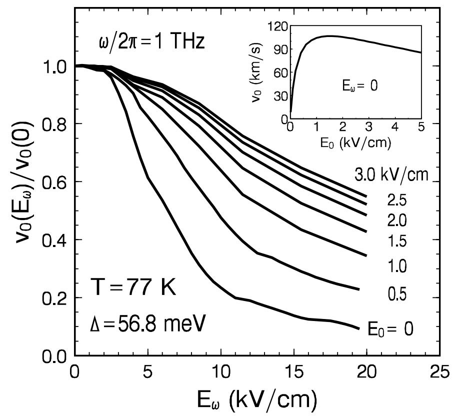

Fig. 1. Average drift velocity $v_0(E_\omega)$ normalized by its value at $E_\omega = 0$, $v_0(0)$, is plotted as a function of the amplitude of the radiation field E_ω at several different strength of the bias dc field $E_0 = 0, 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0 kV/cm, for a GaAs-based superlattice having period $d = 4.8$ nm, miniband width $\Delta = 56.8$ meV, carrier sheet density $N_s = 2.0 \times 10^{15}$ m^{-2} per layer, and low-temperature linear dc mobility (in the absence of the radiation field) $\mu_0 = 10$ m^2/Vs. The lattice temperature is $T = 77$ K, and the frequency of the radiation field is 1 THz. The inset show the v_0–E_0 curve of the system at $T = 77$ K in the absence of the radiation field.

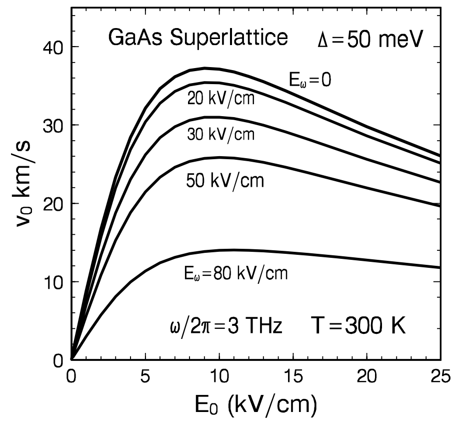

Fig. 2. Average drift velocity v_0 *versus* dc field E_0 under the influence of a radiation electric field of 3 THz frequency with several different amplitude $E_\omega = 0, 20, 30, 50$ and 80 kV/cm in a GaAs-based superlattice having period $d = 4.8$ nm, miniband width $\Delta = 50$ meV, carrier sheet density $N_s = 0.39\times10^{15}$ m^{-2}, and low-temperature linear dc mobility (in the absence of the radiation field) $\mu_0 = 0.15$ m^2/Vs. The lattice temperature $T = 300$ K.

expressions in reference [16], as it should be. Likewise, for very high frequency we have a vanishing s_p and the same $\mathbf{A}_i$, $\mathbf{A}_p$ and W as those without a radiation field, due to the fact that J_n^2 behaves like $\omega^{-4|n|}$. This indicates that a very-high frequency radiation field has no influence on intraband carrier transport. The present balance equation approach is devised for use in the far-infrared or THz regime of the electromagnetic waves, in which the frequency of the radiation field is high enough such that the identification of $\langle \mathbf{V}_0 \rangle$ as the average drift velocity and the approximation (28) are valid, yet the radiation field can strongly affect the transport behavior of the carriers in semiconductors. We expect that the balance equations developed here apply to the time averaging, hot-electron transport driven by an intense radiation field having frequency of order of magnitude or higher than 1 THz.

4 Superlattice miniband transport subject to an intense THz field

As a check of the formulation we apply the above equations to examine the effect of an intense THz radiation on superlattice miniband transport. Consider a GaAs-based planar superlattice in which electrons travel along its growth axis (the z-direction) through the (lowest) miniband formed by periodically spaced potential wells and barriers of finite height. The electron energy dispersion of the system can be written as the sum of the transverse energy $\varepsilon_{\mathbf{k}_\parallel} = k_\parallel^2/2m$, ($\mathbf{k}_\parallel \equiv (k_x, k_y)$) and a tight-binding-type miniband energy $\varepsilon(k_z)$ related to the longitudinal motion:

$$\varepsilon(\mathbf{k}_\parallel, k_z) = \varepsilon_{\mathbf{k}_\parallel} + \varepsilon(k_z) \tag{42}$$

with

$$\varepsilon(k_z) = \frac{\Delta}{2}(1 - \cos k_z d), \tag{43}$$

where d is the superlattice period, $-\pi/d < k_z \le \pi/d$, and Δ is the miniband width. In view of the axial symmetry of the system, when both the dc electric field $\mathbf{E}_0$ and the sinusoidal high-frequency field $\mathbf{E}_\omega$ are polarized along the superlattice growth axis, the carrier drift motion, *i.e.* $\mathbf{p}_d$ and $\mathbf{v}_d$, is in the z direction.

We have carried out numerical calculations in this configuration for two GaAs-based quantum-well superlattices respectively at lattice temperature $T = 77$ and 300 K. We consider the elastic scattering due to randomly distributed background charged impurities, the scattering due to the longitudinal and transverse acoustic phonons (piezoelectric and deformation-potential interactions with electrons), and the scattering due to polar-optic-phonons (Fröhlich coupling with electrons). All the material constants used in the calculation are typical values of bulk GaAs which are the same as those given in reference [18].

Figure 1 shows the effect of a radiation field of 1 THz at lattice temperature $T = 77$ K on the average drift velocity of a GaAs-based superlattice having period $d = 4.8$ nm, miniband width $\Delta = 56.8$ meV, carrier sheet density $N_s = 2.0 \times 10^{15}$ m^{-2} per layer, and low-temperature linear dc mobility (in the absence of radiation field) $\mu_0 = 10$ m^2/Vs. For fixed strength of the dc field E_0, the average drift velocity $v_0(E_\omega)$ normalized by its value in the absence of the THz field, $v_0(0)$, decreases with increasing strength of the radiation field. This dc current suppression is stronger at lower dc field than at higher dc field, and in the limit of $E_0 \to 0$, the dc current at $E_\omega = 20$ kV/cm is less than 0.1 of its value at zero E_ω.

In Figure 2 we plot the average drift velocity v_0 *versus* the dc field E_0 for another GaAs superlattice at lattice temperature $T = 300$ K under the influence of a 3 THz radiation field having several different amplitudes $E_\omega = 0, 20, 30, 50$ and 80 kV/cm. The parameters of the superlattice are: period $d = 4.8$ nm, miniband width $\Delta = 50$ meV, carrier sheet density $N_s = 0.39 \times 10^{15}$ m^{-2}, and low-temperature linear dc mobility (in the absence of radiation field) $\mu_0 = 0.15$ m^2/Vs. In the presence of an intense radiation field, the v_0–E_0 curves, which exhibit negative differential mobility, shift downward, in reasonable agreement with the experimental results of Schomburg *et al.* [6] and Winnerl *et al.* [7].

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